LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **PHYSICS**

FIRST SEMESTER – **APRIL 2014**

PH 1820 - MATHEMATICAL PHYSICS - I

Date : 07/04/2014 Dept. No. Max. : 100 Marks
Time : 09:00-12:00

PART A

Answer **ALL** the questions (10 × 2 = 20)

1) Write down the algorithm for the Regula- Falsi method.
2) Write down the formulae for the Euler modified method.
3) Determine and graph the loci represented by -1< Re z < 1.
4) Evaluate the complex line integral $\int dz/z^2$ around the closed loop C: |z| = 1.
5) Distinguish between a set of linearly independent and linearly dependent vectors.
6) List the properties of a scalar.
7) Write down the transformation equations and its inverse between the Cartesian coordinates x,y,z and the spherical polar coordinates r,θ, ϕ.
8) Define the contravariant and covariant vectors of rank two by their transformation properties.
9) Define the gamma and the beta functions.
10) Use the Rodrigue's formula for the Legendre polynomial to evaluate the 3rd order polynomial.

PART – B

Answer any **FOUR** questions (4 × 7.5 = 30)

11) Using Euler's method, obtain the solution of $\frac{dy}{dx} = x - y$ with y (0) = 1 at x = 0 (0.1) 0.4.
12) Show that the function v (x,y) = -sin x sinh y is harmonic. Construct the corresponding analytic function
f (z) = u (x, y) + i v (x, y).
13) Prove that
a) the eigenvalues of a Hermitian matrix are real
b) any two eigenvectors belonging to distinct eigenvalues are mutually orthogonal to each other.
c) Prove that the eigenvalues of a unitary matrix have a unit modulus.

14) Show that velocity and acceleration are contravariant vectors and that the gradient of a scalar field is a covariant vector.

15) Assuming the recurrence formulas (i) $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$ and (ii) $\frac{d}{dx}[x^{-n} J_n(x)] = -x^{-n} J_{n+1}(x)$ establish $J_{\frac{3}{2}}(x) = \frac{1}{x} J_{\frac{1}{2}}(x) - J_{-\frac{1}{2}}(x)$, where $J_n(x)$ is the Bessel function of the first kind.

PART – C

Answer any **FOUR** questions (4 × 12.5 = 50)

16) Use Gauss- Seidel iterative method to solve the system of equations x + y + z = 2; 2x + y – 3z = -3 ; x – 2y + 4z = 8. Verify your result by Gauss elimination method without pivoting.

17) a) Using the contour integration, evaluate the following real integral, $\int_0^\infty \frac{dx}{(1+x^2)^3}$

(b) Evaluate the following integral using Cauchy's integral formula $\int_c \frac{4-3z}{z(z-1)(z-2)}$ dz, where C is the circle |Z |= 3/2.

18) a) Determine the eigenvalues and the normalized eigenvectors of the matrix A = $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ and the algebraic and the geometric multiplicities.

(b) If λ_i (i = 1,2,3) are the eigenvalues of the matrix B = $\begin{pmatrix} 1 & 0 & -2 \\ -3 & 4 & 1 \\ 2 & 1 & 3 \end{pmatrix}$, then determine $\sum \lambda_i^3$.

19) What is a fully antisymmetric tensor? Express (a) the vector product of two vectors and (b) the commutation relation between the components of angular momentum in quantum mechanics by using the fully antisymmetric tensor of rank three

20) Solve the Legendre differential equation $(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$ by the power series method.